

# Modified holographic Ricci dark energy model and statefinder diagnosis in flat universe.

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## Abstract

Evolution of the universe with modified holographic Ricci dark energy model is considered. Dependency of the equation of state parameter and deceleration parameter on the redshift and model parameters are obtained. It is shown that the density evolution of both the non-relativistic matter and dark energy are same until recent times. The evolutionary trajectories of the model for different model parameters are obtained in the statefinder planes,  $r-s$  and  $r-q$  planes. The present statefinder parameters are obtained for different model parameter values, using that the model is differentiated from other standard models like  $\Lambda$ CDM model etc. We have also shown that the evolutionary trajectories are depending on the model parameters, and at past times the dark energy is behaving like cold dark matter, with equation of state equal to zero.

**Keywords:** Dark energy, Holographic model, Statefinder diagnostic, Cosmological evolution.

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# 1 Introduction

Observations of distant type Ia supernovae (SNIa) and cosmic microwave background anisotropy have shown that the present universe is accelerating [1]. This expansion may be driven by a component with negative pressure, called dark energy. The simplest model of dark energy is the cosmological constant  $\Lambda$  which can fit the observations in a fair way [2, 3], whose equation of state is  $\omega_\Lambda = -1$ . during the evolution of the universe. However there are two serious problems with cosmological constant model, namely the fine tuning and the cosmic coincidence [4]. To solve these problems different dynamic dark energy models have been proposed, with varying equation of state during the expansion of the universe. Holographic dark energy (HDE) is one among them [5, 6, 7]. HDE is constructed based on the holographic principle, that in quantum gravity, the entropy of a system scales not with its volume but with its surface area  $L^2$ , analogically the cosmological constant in Einstein's theory also is inverse of some length squared. It was shown that [5] in effective quantum field theory, the zero point energy of the system with size  $L$  should not exceed the mass of a black hole with the same size, thus  $L^3\rho_\Lambda \leq LM_P^2$ , where  $\rho_\Lambda$  is the quantum zero-point energy and  $M_P = 1/\sqrt{8\pi G}$ , is the reduced Plank mass. This inequality relation implies a link between the ultraviolet (UV) cut-off, defined through  $\rho_\Lambda$  and the infrared (IR) cut-off encoded in the scale  $L$ . In the context of cosmology one can take the dark energy density of the universe  $\rho_x$  as the same as the vacuum energy, i.e.  $\rho_x = \rho_\Lambda$ . The largest IR cut-off  $L$  is chosen by saturating the inequality, so that the holographic energy density can be written as

$$\rho_x = 3c^2 M_P^2 L^{-2} \quad (1)$$

where  $c$  is numerical constant. In the current literature, the IR cut-off has been taken as the Hubble horizon [6, 7], particle horizon and event horizon [7] or some generalized IR cut off [8, 9, 10]. The HDE models with Hubble horizon or particle horizon as the IR cut-off, cannot lead to the current accelerated expansion [6] of the universe. When the event horizon is taken as the length scale, the model is suffered from the following disadvantage. Future event horizon is a global concept of space-time. On the other hand density of dark energy is a local quantity. So the relation between them will pose challenges to the concept of causality. These leads to the introduction new HDE, where the length scale is given by the average radius of the Ricci scalar curvature,  $R^{-1/2}$ .

The holographic Ricci dark energy model introduced by Granda and Oliveros [11] based on the space-time scalar curvature, is fairly good in fitting with the observational data. This model have the following advantages. First, the fine tuning problem can be avoided in this model. Moreover, the presence of event horizon is not presumed in this model, so that the causality problem can be avoided. The coincidence problem can also be solved effectively in this model. Recently a modified form of Ricci dark energy was studied [12] in connection with the dark matter interaction, and analyses the model using  $Om$  diagnostic. In this paper we have considered the evolution of the universe in Modified Holographic Ricci Dark Energy (MHRDE) model and obtain the statefinder parameters to discriminate this model with other standard dark energy models.

Statefinder parameters is a sensitive and diagnostic tool used to discriminate various dark energy models. The Hubble parameter  $H$  and deceleration parameter  $q$  alone cannot discriminate various dark energy models because of the degeneracy on these parameters. Hence Sahni et al.

[13] introduces a set of parameters  $\{r, s\}$  called statefinder parameters, defined as,

$$r = \frac{\ddot{a}}{aH^3}, \quad s = \frac{r - \Omega_{total}}{3(q - \Omega_{total})/2}, \quad (2)$$

where  $a$  is the scale factor of the expanding universe and  $\Omega_{total}$  is the total energy density containing dark energy, energy corresponds to curvature and also matter (we are neglecting the radiation part in our analysis). In general statefinder parameter is a geometrical diagnostic such that it depends upon the expansion factor and hence on the metric describing space-time. The  $r - s$  plot of dark energy models can help to differentiate and discriminate various models. For the well known  $\Lambda$ CDM model, the  $r - s$  trajectory is corresponds to fixed point, with  $r = 1$  and  $s = 0$  [13]. The cosmological behavior of various dark models including holographic dark energy model, were studied and differentiated in the recent literature using statefinder parameters [14, 15, 16, 17].

The paper is organized as follows. In section 2, we have studied the cosmological behavior of the MHRDE model and in section 3 we have considered the statefinder diagnostic analysis followed by the conclusions in section 4.

## 2 The MHRDE model

The universe is described by the Friedmann-Robertson-Walker metric given by

$$ds^2 = -dt^2 + a(t)^2 \left( \frac{dr^2}{1 - kr^2} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \right), \quad (3)$$

where  $(r, \theta, \phi)$  are the co-moving coordinates,  $k$  is the curvature parameter with values,  $k = 1, 0, -1$  for closed, flat and open universes respectively and  $a(t)$  is the scale factor, with  $a_0 = 1$ , is taken as its present value. The Friedmann equation describing the evolution of the universe is

$$H^2 + \frac{k}{a^2} = \frac{1}{3} \sum_i \rho_i, \quad (4)$$

where we have taken  $8\pi G = 1$ , the summation includes the energy densities of non-relativistic matter and dark energy, i.e.  $\sum_i \rho_i = \rho_m + \rho_x$ . The modified holographic Ricci dark energy can be expressed by taking the IR cutoff with the modified Ricci radius in terms of  $\dot{H}$  and  $H^2$  as [11, 12]

$$\rho_x = \frac{2}{\alpha - \beta} \left( \dot{H} + \frac{3\alpha}{2} H^2 \right), \quad (5)$$

where  $\dot{H}$  is the time derivative of the Hubble parameter,  $\alpha$  and  $\beta$  are free constants, the model parameters. Chimento et. al. studied this type of dark energy in interaction with the dark matter with Chaplygin gas [12], but our analysis is mainly concentrated on the cosmological evolution of MHRDE and analyses with statefinder diagnostic. Substituting the dark energy density as the MHRDE in the Friedmann equation, and changing the variable from cosmic time  $t$  to  $x = \ln a$  we get

$$H^2 + \frac{k}{a^2} = \frac{\rho_m}{3} + \frac{1}{3(\alpha - \beta)} \frac{dH^2}{dx} + \frac{\alpha}{\alpha - \beta} H^2 \quad (6)$$

Introducing the normalized Hubble parameter as  $h = H/H_0$  and  $\Omega_k = -k/H_0^2$ , where  $H_0$  is the Hubble parameter for  $x = 0$ , the above equation become,

$$h^2 - \Omega_{k0}e^{-2x} = \Omega_{m0}e^{-3x} + \frac{1}{3(\alpha - \beta)} \frac{dh^2}{dx} + \frac{\alpha}{\alpha - \beta} h^2, \quad (7)$$

where  $\Omega_{m0} = \rho_{m0}/3H_0^2$  is the current density parameter of non-relativistic matter ( we will take 0.27 as its values for our analysis throughout.) with current density  $\rho_{m0}$  and  $\Omega_{k0}$  is the present relative density parameter of the curvature. We will consider only flat universe, where  $\Omega_{k0} = 0$  in our further analysis. Solving the first order differential equation (7) we obtain the dimensionless Hubble parameter  $h$  as,

$$h^2 = \Omega_{m0}e^{-3x} + \frac{\alpha - 1}{1 - \beta} \Omega_{m0}e^{-3x} + \left[ \frac{(\alpha - \beta)\Omega_{m0}}{\beta - 1} + 1 \right] e^{-3\beta x}. \quad (8)$$

Comparing this with the standard Friedman equation, the dark energy density can be identified as

$$\Omega_x = \frac{\alpha - 1}{1 - \beta} \Omega_{m0}e^{-3x} + \left[ \frac{(\alpha - \beta)\Omega_{m0}}{\beta - 1} + 1 \right] e^{-3\beta x} \quad (9)$$

This shows that similar to the result obtained in references [8, 15] for Ricci dark energy, the MHRDE density has one part which evolves like non-relativistic matter ( $\sim e^{-3x}$ ) and the other part is slowly increasing with the decrease in redshift. The pressure corresponding the dark energy can be calculated as,

$$p_x = -\Omega_x - \frac{1}{3} \frac{d\Omega_x}{dx} = [(\alpha - \beta)\Omega_{m0} + \beta - 1] e^{-3\beta x} \quad (10)$$

Form the conservation equation, we can obtain the corresponding equation of state parameter for the flat universe, using equation (9) as,

$$\omega_x = -1 - \frac{1}{3} \frac{d \ln \Omega_x}{dx} = -1 + \left\{ \frac{(\alpha - 1)\Omega_{m0} + \beta [(1 - \beta) - (\alpha - \beta)\Omega_{m0}] e^{3(1-\beta)x}}{(\alpha - 1)\Omega_{m0} + [(1 - \beta) - (\alpha - \beta)\Omega_{m0}] e^{3(1-\beta)x}} \right\} \quad (11)$$

This equation of state implies the possibility of transit form  $\omega_x > -1$  to  $\omega_x < -1$ , corresponds to the phantom model [18, 19] for suitable model parameter values. Recent observational evidences shows that the dark energy equation of state parameter can crosses the value -1 [20]. In a universe dominated with MHRDE, where the contribution from the non-relativistic matter behavior term is negligible in the dark energy density, the equation state parameter become,

$$\omega_x = -1 + \beta \quad (12)$$

So if  $\beta$  is less than zero, the equation of state can crosses the phantom divide. In the far future of the universe, when redshift  $z \rightarrow -1$  also, the equation of state parameter reduces to the form given in equation (12). So the behavior of the dark energy is depending strongly on the model parameter  $\beta$ .

We have plotted the evolution of the equation of state parameter of MHRDE with redshift in figure 1, using the best fit values of the model parameters  $\alpha$  and  $\beta$  as [12]  $(\alpha, \beta) = (1.01, 0.15)$  and  $\Omega_{m0}=0.27$ . The evolution of  $\omega_x$  of dark energy shows that in the remote past of the universe, that is at large redshift, the equation of state parameter is near zero, implies that the dark energy behaves like the cold dark matter in the remote past. The plot also shows that at far future of the universe as  $z \rightarrow -1$ , the equation of state parameter approaches a saturation value. The

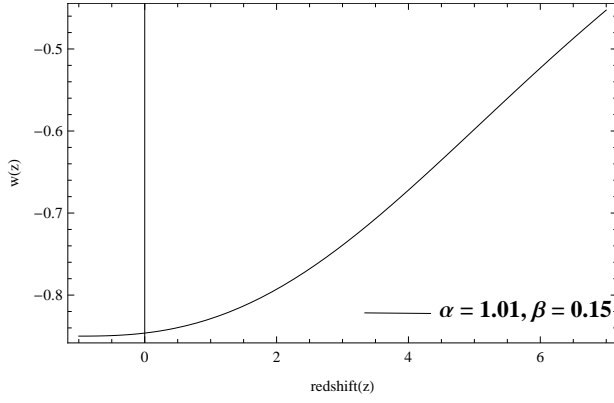


Figure 1: Evolution of equation of state parameter  $\omega_x$  with redshift  $z$  for the best fit values  $\alpha = 1.01$  and  $\beta = 0.15$ .

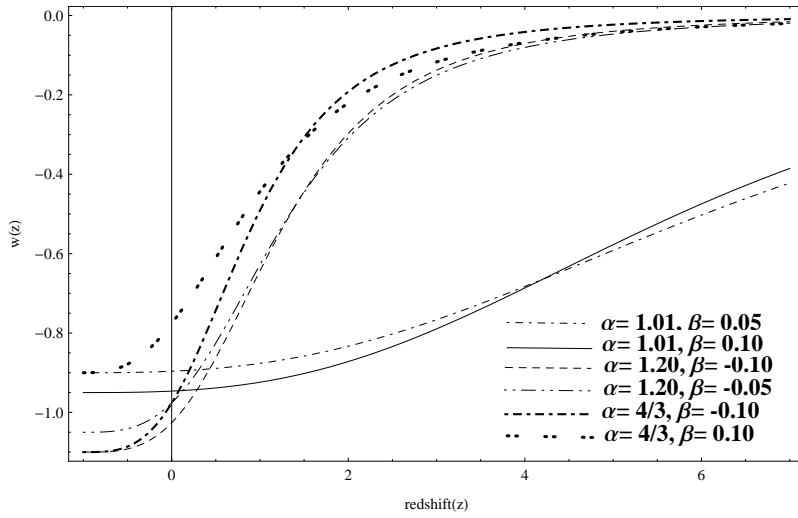


Figure 2: Evolution of the equation of state parameter for other values of  $\alpha$  and  $\beta$

present value of the equation of state parameter according to this plot is negative, and is around  $\omega_x = -0.7$ .

For other values of the model parameters  $(\alpha, \beta)$  [12] as  $(\alpha, \beta) = (1.01, 0.05)$ ,  $(1.01, 0.1)$ ,  $(1.2, -0.05)$ ,  $(1.2, -0.1)$ ,  $(4/3, 0.1)$  and  $(4/3, -0.1)$  the behavior of the equation of state parameter is given figure 2. For a given value of  $\alpha$  the saturation value of  $\omega_x$  in the future universe decreases as  $|\beta|$  increases.

Figure 2 shows that irrespective of the values of the parameters  $(\alpha, \beta)$ , the equation of state parameter is negative at present times implies that the present universe is accelerating, and also in the remote past at high redshift  $\omega_x \rightarrow 0$ , indicate that MHRDE behaves like cold dark matter in the past stages of the universe. For negative values of  $\beta$  the equation of state parameter crosses -1, in that case it can be classified as quintom [22] dark energy and for the case  $\omega_x < -1$  the universe will evolve into a phantom energy dominated epoch [21].

In figure 3, we have shown a comparison of the evolution of non-relativistic matter density and MHRDE density in logarithmic scale. Here we have neglected phase transitions, transitions

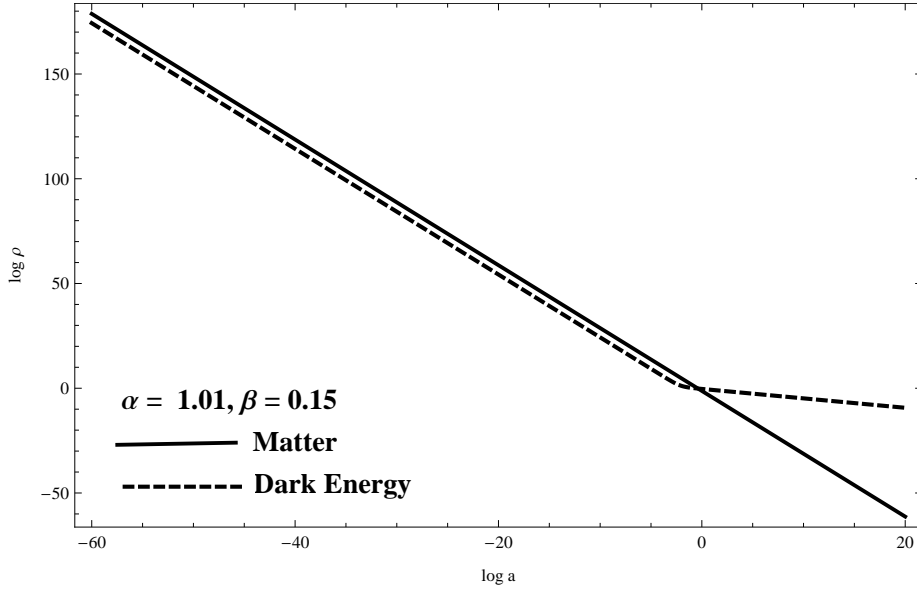


Figure 3: *Evolution of non-relativistic matter density and MHRDE density in log.scale*

from non-relativistic to relativistic particles at high temperatures and new degrees of freedom etc. It is expected that these would not make much qualitative difference in the result. The plot shows that in the present model the densities of non-relativistic matter and dark energy were comparable with each other in the past universe that is at high redshift. The acceleration began at low redshifts, which solves the coincidence problem.

The deceleration parameter  $q$  for the MHRDE model can be obtained using the relation

$$q = -\frac{\dot{H}}{H^2} - 1. \quad (13)$$

This equation can be expressed in terms of the dimensionless Hubble parameter  $h$  as

$$q = -\frac{1}{2h^2} \frac{dh^2}{dx} - 1 \quad (14)$$

Using equation (8) the above equation can be written as

$$q = \frac{\left(\frac{\alpha-\beta}{1-\beta}\right) \Omega_{m0} e^{-3x} + \left[\frac{(\alpha-\beta)\Omega_{m0}}{\beta-1} + 1\right] (3\beta-2) e^{-3\beta x}}{2 \left[\left(\frac{\alpha-\beta}{1-\beta}\right) \Omega_{m0} e^{-3x} + \left(\frac{(\alpha-\beta)\Omega_{m0}}{\beta-1} + 1\right) e^{-3\beta x}\right]} \quad (15)$$

This equation shows the dependence of the deceleration parameter on the model parameters  $\alpha$  and  $\beta$ . As an approximation, if we neglect the contribution from the first terms in both numerator and denominator (since they are negligibly small) the deceleration parameter will become  $q = (3\beta - 2)/2$ . Which shows, as  $\beta$  increases from zero, the parameter  $q$  increases from -1, that is the universe enters the acceleration phase at successively later times. In figure 4 and figure 5 we have plotted the evolution of the deceleration parameter with redshift. Figure 4 is for the best fitting model parameters  $\alpha=1.01$ ,  $\beta=0.15$  and figure 5 is for the remaining model parameter values. The plots show that at large redshift, the deceleration parameter approaches 0.5. The universe is entering the acceleration in the recent past at  $z < 1$ . The plot also shows that as  $\alpha$  increases, the entry to the accelerating phase is occurring at relatively lower values of redshift, that is the universe entering the accelerating phase at relatively later times as the parameter  $\alpha$

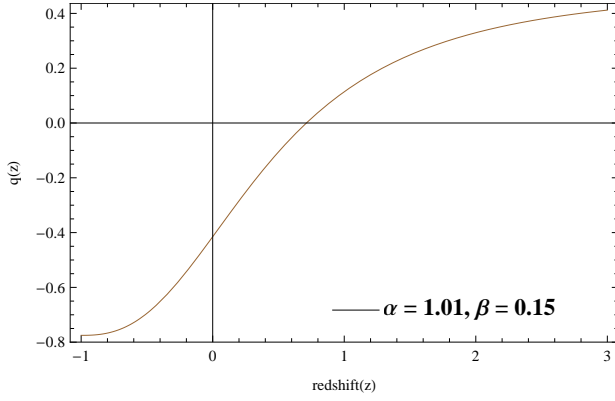


Figure 4: *Evolution of deceleration parameter  $q$  for the best fit model parameters  $\alpha=1.01$  and  $\beta=0.15$*

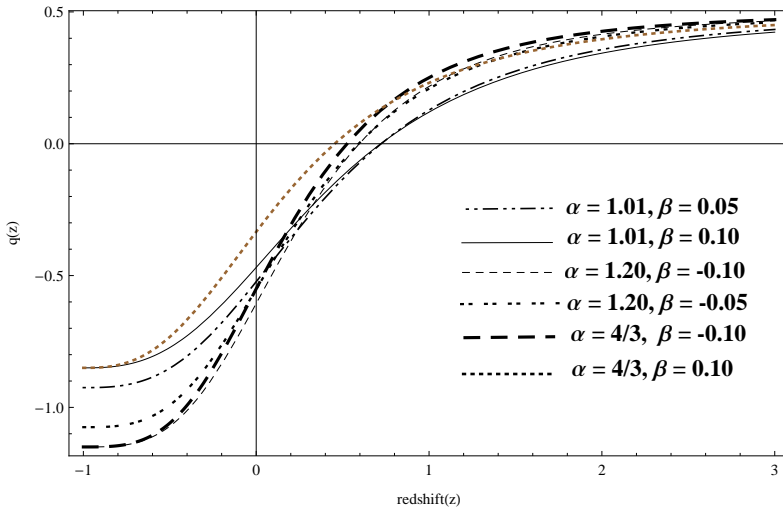


Figure 5: *Evolution of deceleration parameter for other values of the model parameter.*

increases. The transition of the universe from deceleration to the accelerating phase is occurred at the the redshift  $Z_T = 0.76$ , for the best fit model parameters . For comparison the combined analysis of SNe+CMB data with  $\Lambda$ CDM model gives the range  $Z_T(\Lambda\text{CDM}) = 0.50 - 0.73$  [20, 24]. For taking consideration of the entire model parametric range, the transition to the accelerating phase can be obtained, as in figure 5 as  $Z_T(\text{MHRDE})=0.50 - 0.76$ . The comparison of the two ranges shows that in the MHRDE model the universe entering the accelerating expansion phase earlier than in the  $\Lambda$ CDM model. The present value of the deceleration parameter for the best fit model parameters  $\alpha=1.01$ ,  $\beta=0.15$  is  $q_0 = -0.45$  as from figure 4.

### 3 Statefinder diagnostic

We have calculated the statefinder parameters  $r$  and  $s$ , as defined earlier in equation (2). Statefinder parameters can provide us with a diagnosis which should unambiguously probe the properties of various classes of dark energy models. Equation (2) for  $r$  and  $s$  can be rewrite in terms of  $h^2$  as,

$$r = \frac{1}{2h^2} \frac{d^2 h^2}{dx^2} + \frac{3}{2h^2} \frac{dh^2}{dx} + 1 \quad (16)$$

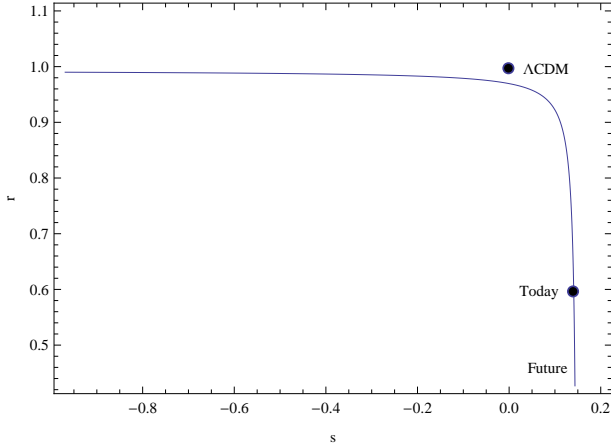


Figure 6: *Evolutionary trajectory in the  $r - s$  plane for MHRDE model for the best fit values of the model parameters  $\alpha=1.01$ ,  $\beta=0.15$ . The black spot on the top right corner corresponds to  $r = 1$ ,  $s = 0$  the  $\Lambda$ CDM model. The today's point corresponds to  $r=0.59$ ,  $s=0.15$*

and

$$s = - \left\{ \frac{\frac{1}{2h^2} \frac{dh^2}{dx^2} + \frac{3}{h^2} \frac{dh^2}{dx}}{\frac{3}{2h^2} \frac{dh^2}{dx} + \frac{9}{2}} \right\} \quad (17)$$

On substituting the relation for  $h^2$  from equation (8), the above equations for a flat universe (in which  $\Omega_k = 0$ ) become

$$r = 1 + \left\{ \frac{9\beta(\beta - 1) \left( \frac{(\alpha - \beta)\Omega_{m0}}{\beta - 1} + 1 \right) e^{-3\beta x}}{2 \left[ \Omega_{m0}e^{-3x} + \left( \frac{\alpha - 1}{1 - \beta} \right) \Omega_{m0}e^{-3\beta x} + \left( \frac{(\alpha - \beta)\Omega_{m0}}{\beta - 1} + 1 \right) e^{-3\beta x} \right]} \right\} \quad (18)$$

and  $s$  is become,

$$s = - \left\{ \frac{\beta(\beta - 1) \left( \frac{(\alpha - \beta)\Omega_{m0}}{\beta - 1} + 1 \right) e^{-3\beta x}}{\left[ \frac{(\alpha - \beta)\Omega_{m0}}{\beta - 1} + 1 \right] (1 - \beta)e^{-3\beta x}} \right\} = \beta \quad (19)$$

From equations (18) and (19), it is evident that  $r = 1$ ,  $s = 0$  if  $\beta = 0$  and no matter what value  $\alpha$  is, and this point in the  $r - s$  plane is corresponds to the  $\Lambda$ CDM model. This point is a very fixed point, thus statefinder diagnostic fails to discriminate between  $\Lambda$ CDM model and MHRDE model for the model parameter value  $\beta = 0$ . Since  $s$  is a constant for flat universe in this model, the trajectory in the  $r - s$  plane is a vertical segment, with constant  $s$  during the evolution of the universe, while  $r$  is monotonically decreasing form 1, if  $\beta$  is positive and monotonically increasing if  $\beta$  is assuming negative values. For a simple understanding, let us assume that  $\Omega_m$  contribution is negligible small, when dark energy is dominating, then the equation (18) reduces to

$$r = 1 + \frac{9\beta(\beta - 1)}{2} \quad (20)$$

In this case for  $\beta=0.05$ ,  $0.1$  and  $0.15$  the corresponding values of  $r$  are  $0.79$ ,  $0.60$ ,  $0.43$  respectively. Bur when  $\beta$  assumes the negative values  $-0.05$  and  $-0.10$ , the corresponding values of  $r$  become  $1.24$  and  $1.5$  respectively. So at the outset the MHRDE model gives a  $r - s$  trajectory, as  $r$  starting form 1 and due to evolution of the universe the  $r$  will decreases to  $1 - \frac{9\beta(1-\beta)}{2}$  if  $\beta$  is positive and increases to  $1 + \frac{9\beta(\beta-1)}{2}$ , if  $\beta$  is negative.

The  $r - s$  evolutionary trajectory in the MHRDE model in flat universe for the best fit model parameters  $\alpha=1.01$  and  $\beta=0.15$ , is given in figure 6. In this plot as the universe expands, the



trajectory in the  $r - s$  plane starts from left to right. The standard  $\Lambda$ CDM model is corresponds to  $r = 1, s = 0$  is denoted. In this model the parameter  $r$  first decrease very slowly with  $s$ , then after around  $s=0$   $r$  decreases steeply. The today's value of the statefinder parameter ( $r_0=0.59, s_0=0.15$ ) is denoted in the plot.

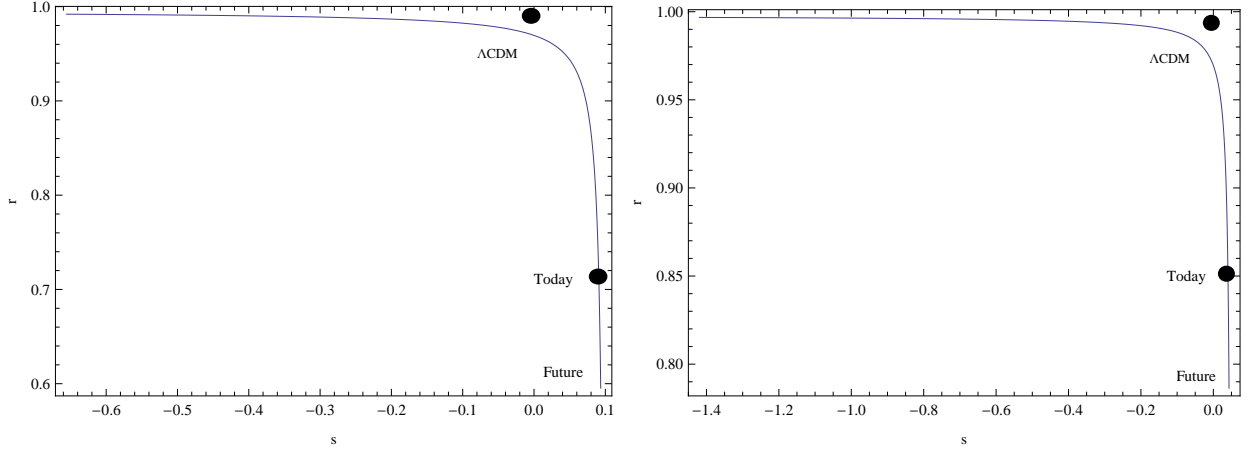


Figure 7: the first plot is for  $\alpha, \beta = 1.01, 0.10$  and the second plot is for  $\alpha, \beta = 1.01, 0.05$ . The black spot on the top right corner corresponds to  $\Lambda$ CDM model, the present state of the evolution is denoted as today's point.

For other model parameters, the  $r - s$  plots are given in figure 7. These plots also shows the same behavior of figure 6, but the separation between  $\Lambda$ CDM model and MHRDE model in the  $r - s$  plane is increasing as  $\beta$  increases. The respective today's universe corresponds  $r_0, s_0 = 0.71, 0.1$  and  $r_0, s_0 = 0.85, 0.05$ .

For negative values of  $\beta$ , the evolutionary characteristics is plotted in figure 8 for model parameters  $\alpha, \beta = 1, 2, -0.10; 4/3, -0.10$ . Here also the evolution in the  $r - s$  plane is from left to right. In this case the behavior is different from that for the positive  $\beta$  value, in the sense that as  $s$  increases  $r$  is increasing to values greater than one. The increase is very slowly at first then increases steeply as the universe evolves. The today's value in these cases are  $r_0=1.325, s_0=-0.10$  when  $(\alpha, \beta) = (1.2, -0.10)$  and  $r_0=1.321, s_0=-0.10$  for  $(\alpha, \beta) = (4/3, -0.10)$  respectively. The difference between MHRDE model for these model parameters and  $\Lambda$ CDM can be noted.

The statefinder diagnostic can discriminate this model with other models. As example, for the quintessence model the  $r - s$  trajectory is lying in the region  $s > 0, r < 1$  and for Chaplygin gas the trajectory is in the region  $s < 0, r > 1$ . Holographic dark energy with the future event horizon as IR cutoff, starts its evolution from  $s = 2/3, r = 1$  and ends on at  $\Lambda$ CDM model fixed point in the future [16, 27].

In order to confirm the  $r - s$  behavior of MHRDE model, we have plotted the behavior in  $r - z$  plane, in figure 9. For MHRDE model, the  $r$  value is commencing from 1 irrespective of the values of  $\alpha$  and  $\beta$  at remote past and as the universe evolves,  $r$  is decreasing, if  $\beta$  is positive and increasing, if  $\beta$  is negative.

We have studied the evolutionary behavior in the  $r - q$  plane also. For positive value of  $\beta$  the plot is shown in figure 10 for the standard values  $\alpha = 1.01, \beta = 0.15$ . The figure shows that both  $\Lambda$ CDM model and MHRDE model commence evolving from the same point in the

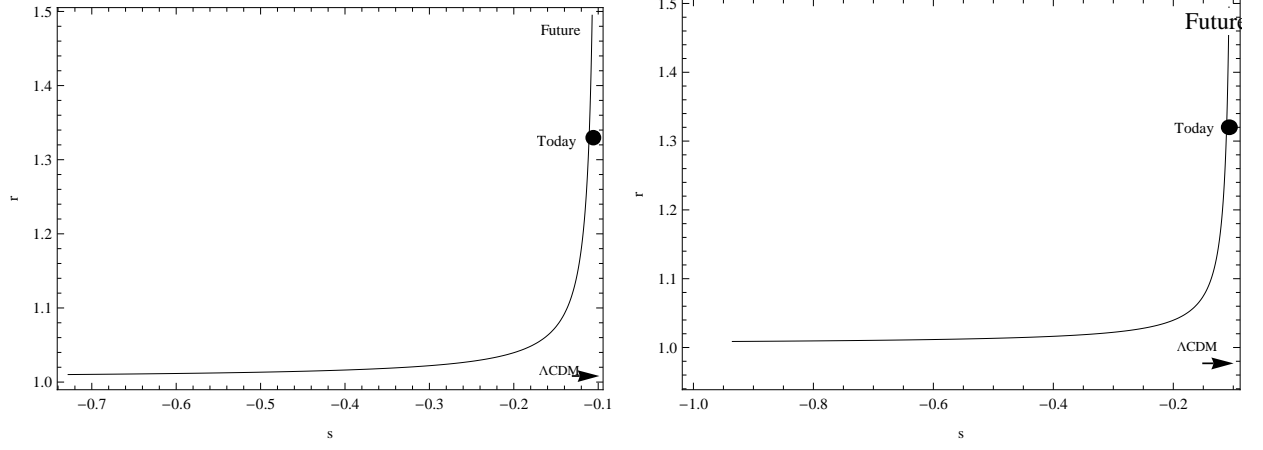


Figure 8:  $r - s$  plots for model parameters  $\alpha, \beta = 1.2, -0.10; 4/3, -0.10$ . the arrow in lower left corner of the panel shows the evolution towards  $\Lambda$ CDM model. The present position of the evolution is denoted as today's point.

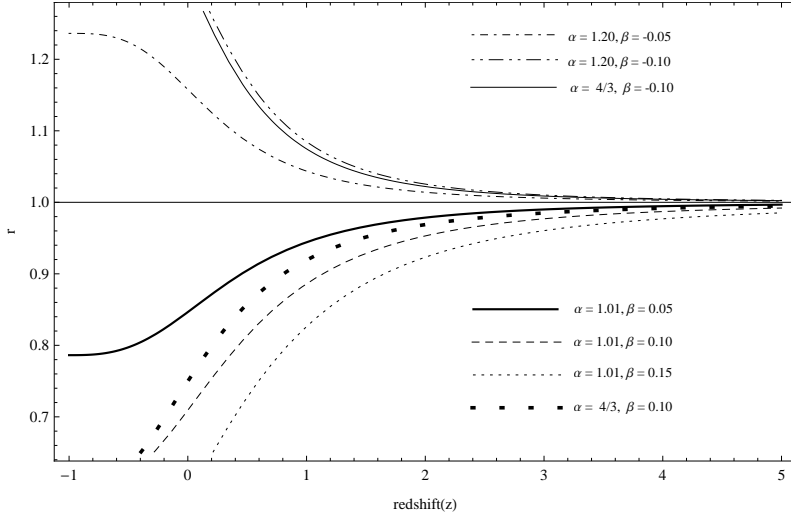


Figure 9:  $r - z$  plot, all model parameters. Shows that for positive values of  $\beta$   $r$  is decreasing from 1, but for negative values of  $\beta$  the value of  $r$  is increases from 1.

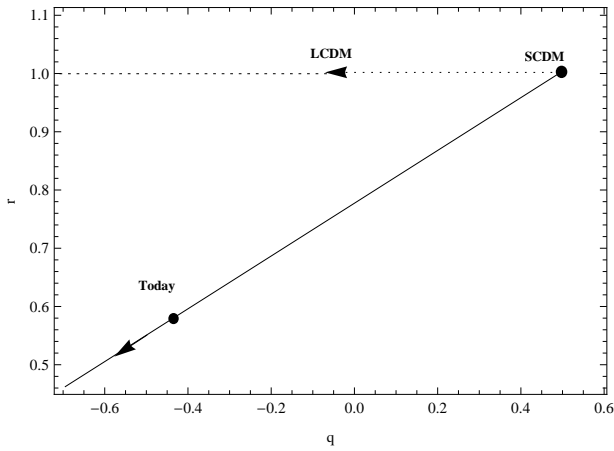


Figure 10: Evolutionary trajectory in the statefinder  $r - q$  plane with  $\alpha = 1.01$  and  $\beta = 0.15$ . The solid line represents the MHRDE model, and the dashed line the  $\Lambda$ CDM (denoted as LCDM model in the plot) as comparison. Location of today's point is  $(0.59, -0.45)$ .

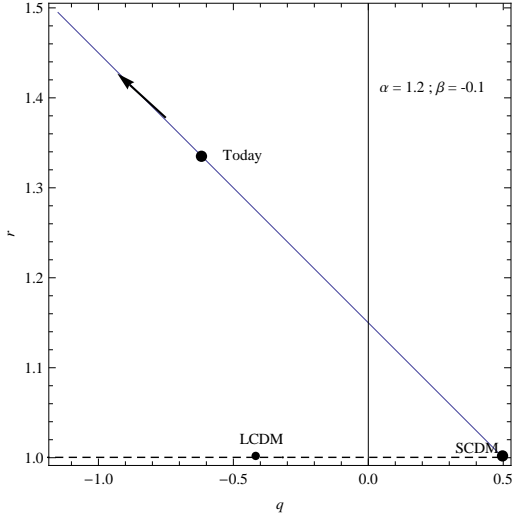


Figure 11: *Evolutionary trajectory in the  $r-q$  plane with  $\alpha=1.2$ ,  $\beta=-0.10$ . The present position is denoted. The dashed shows the evolution of  $\Lambda$ CDM (denoted as LCDM model in the plot) model from right to left.*

past corresponds to  $r = 1$ ,  $q = 0.5$ , which corresponds to a matter dominated SCDM universe. In  $\Lambda$ CDM model the trajectory will end their evolution at  $q = -1$ ,  $r = 1$  which corresponds to de Sitter model, while in MHRDE model the behavior is different from this. The statefinder trajectory in holographic dark energy model with future event horizon has the same starting point and the same end point as  $\Lambda$ CDM model [25, 26]. Thus MHRDE model is also different form holographic dark energy with event horizon form the statefinder viewpoint.

For negative values  $\beta$  the plot is as given in figure 11. The evolution of the trajectory is starting from left to right. Note that the  $r$  value is at the increase from one as the universe evolves. It is evident from the plot that the present position of the model corresponds to  $r_0=1.325$  and  $q_0 = -0.63$ .

## 4 Conclusions

We have studied the modified holographic Ricci dark energy (MHRDE) in flat universe, where the IR cutoff is given by the modified Ricci scalar, and the dark energy become  $\rho_x = 2(\dot{H} + 3\alpha H^2/2)/(\alpha - \beta)$  where  $\alpha$  and  $\beta$  are model parameters. We have calculated the relevant cosmological parameters and their evolution and also analyzed the model form the statefinder view point for discriminating it from other models. The importance of the model is that it depends on the local quantities and thus avoids the causality problem.

The density of MHRDE is comparable with the non-relativistic matter at high redshift as shown in figure 3 and began to dominate at low redshifts, thus the model is free from the coincidence problem.

The evolution of equation of state parameter is studied. The equation of state parameter

is nearly zero at high redshift, implies that in the past universe MHRDE behaves like cold dark matter. Further evolution of equation of state is strongly depending on the model parameter  $\beta$ . If the  $\beta$  parameter is positive the equation of state is greater than -1. For negative values of  $\beta$ , the equation of state cross the phantom divide  $\omega_x < -1$ .

In this model the deceleration parameter starts form around 0.5 at the early times and and starts to become negative when the redshift  $z < 1$ . . In general we have found that in MHRDE model the universe entering the accelerating phase at times earlier (for allowed range of parameters  $\alpha$  and  $\beta$ ), than in the  $\Lambda$ CDM model. But in particular as the model parameter  $\alpha$  increases, the universe enter the accelerating phase at relatively later times.

We have applied the statefinder diagnostic to the MHRDE and plot the trajectories in the  $r-s$  and  $r-q$  planes. The statefinder diagnostic is a crucial tool for discriminating different dark energy models. The statefinder trajectories are depending on the model parameters  $\alpha$  and  $\beta$ . For positive values of  $\beta$  the  $r$  values will decreases from one and for negative  $\beta$  the  $r$  will increases form one as the universe evolves. The values of  $\alpha$  and  $\beta$  are constrained using observational data in reference [12], the best fit value is  $\alpha=1.01$ ,  $\beta=0.15$ . The present value of  $(r, s)$  can be viewed as a discriminator for testing different dark energy models. For the  $\Lambda$ CDM model statefinder is a fixed point  $r=1$ ,  $s=0$ . For positive values  $\beta$  parameter the  $r-s$  and  $r-q$  plots of MHRDE shows that, the evolutionary trajectories starts form  $r = 1$  and  $q = 0.5$ , in the past universe (for the best fit model parameters), which reveals that the MHRDE is behaving like cold dark matter in the past. The further evolution of MHRDE in the  $r-s$  plane shows that the present position of MHRDE model in the  $r-s$  plane for the best fit parameter is  $r_0=0.59$ ,  $s_0=0.15$  and in the  $r-q$  plane is  $r_0=0.59$ ,  $q_0=-0.45$ . The difference between the MHRDE and  $\Lambda$ CDM models is in the evolution of the equation of state parameter, which is -1 in the  $\Lambda$ CDM model and a time-dependent variable in MHRDE model. A further comparison can be made with the new HDE model [14], which gives the present values  $r_0(HDE) = 1.357$ ,  $s_0(HDE) = -0.102$  and  $r_0(HDE) = 1.357$ ,  $q_0(HDE) = -0.590$ . So in the  $r-s$  plane the distance of the MHRDE model form the  $\Lambda$ CDM fixed point is slightly larger compared to the new HDE model for positive values of  $\beta$  parameter. However in the case of MHRDE model the starting point in  $r-s$  plane and  $r-q$  plane is ( $r = 1, s = 0$  and  $r = 1, q = 0.5$ ) is same as that in the  $\Lambda$ CDM model.

For negative values of the  $\beta$  the  $r-s$  trajectory we have plotted is different compared to that of positive  $\beta$  values. For negative  $\beta$  values the  $r$  value can attains values greater than one as  $s$  increases. The present status of the evolution in the  $r-s$  plane is  $r_0 = 1.325$ ,  $s_0=-0.10$  for model parameters  $\alpha=1.2$ ,  $\beta=-0.10$  and  $r_0=1.321$ ,  $s_0=-0.10$ . The  $r-q$  for  $\alpha=1.2$  and  $\beta=-0.10$  shows that the present state of the MHRDE model is corresponds to  $r_0 = 1.325$  and  $q_0 = -0.63$ . These values shows that the MHRDE model is different form  $\Lambda$ CDM model for the present time when  $\beta$  parameter is negative also. But compared the new HDE model, the present MHRDE model doesn't show much deviation, shows that for negative values the behavior of MHRDE model is almost similar to new HDE model. Irrespective of whether  $\beta$  is positive or negative the MHRDE model is commence to evolve from SCDM model. When  $\beta$  is positive 1 is the maximum value of  $r$ , on the other hand when  $\beta$  is negative 1 is the minimum value of  $r$ . However the exact discrimination of the dark energy models is possible only if we can obtain the present  $r-s$  values in a model independent way form the observational data. It is expected that the future high-precision SNAP-type observations can lead to the present statefinder parameters, which could help us to find the right dark energy models.

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